A Silent Laborer in Medieval Iberian Science:
The function of paper in scientific production

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Abstract. This paper looks at the introduction of paper as a writing support in the scientific production of medieval Iberia through the 11^{th}-13^{th} centuries, focusing on its influence in astronomy and the broader Islamicate scientific tradition. I analyze paper's affordability, manipulability, and durability, and maintain that paper enabled the proliferation of intermediary scientific documentation, essential for observational note-taking, mathematical computation, and collaborative work. With examples from al-Andalus and the Toledan and Alfonsine tables, I argue paper transformed the information processing infrastructure. Despite limited surviving sources, I emphasize the standing questions around the role of paper in Medieval scientific production, and more broadly the material infrastructure of Medieval writing culture.

1. Introduction. Ibn Khaldūn on the cheapening of writing supports

Paper is one of those technologies that, having penetrated so deeply into the modern world, has receded to the background, and this makes it difficult to really acknowledge its importance. Almost by definition, writing requires a support in which to be inscribed. That, along with some sort of inscribing agent (like a pen, or a stylus), are the two basic material technologies required in the process of writing. This can be seen in the earliest instances of a writing culture, with cuneiform script, which used wax tablets and a simple stylus to carve symbols over it. (Figure 1) Wax tablets presented a clear advantage, and at least one important disadvantage. On the one hand, it was a very durable material: Once dried, and especially if it had not undergone a lot of humidity, what is written on it can last for millennia (as shown by the available extant tablets). However, due to its thickness and weight, it presents serious difficulties of transportation. This highlights a more general point: How information can be stored and distributed depends crucially on the materials on which it is recorded. Every material has its own characteristic trade-offs, which exert an influence on how the writer, as well as the reader, can access the information, and how that information can be processed and distributed.

When a new writing support is introduced, the user of the technology has to evaluate the relative trade-offs of the current material against the alternative. One of the key motivations for any choice of writing material are the financial incentives: How much will it cost me to buy a roll of papyrus, versus, say, a bundle of parchment? A more affordable material has a number of important downstream effects. To start with, final products—like codices, or scrolls—also become cheaper, allowing in principle for a wider consumer base. This has been one of the traditional stories about how, and why, paper spread so rapidly during the initial period of

Figure 1. Writing tablet dated ca. 3100-3300 BCE, likely from today's Southern Iraq. The tablet depicts the symbol for beer, along with cuneiform numerical symbols and a sign representing "to eat." © Trustees of the British Museum

the Islamic civilization. Perhaps the most representative text on this respect is by the historian Ibn Khaldūn (d. 1406):

> Originally, copies of scholarly works, governmental correspondence, and diplomas were written on parchment, because there was great prosperity at the beginning of Islam and the works that were written were few. The production of books and writing developed greatly. Government documents and diplomas increased in number. There was not enough parchment for all that. Therefore, al-Faḍl ibn Yaḥyā suggested the manufacture of paper. Afterwards, people used paper in sheets for government and scholarly writings, and its manufacture reached a considerable degree of excellence. (1967, 328)

Central to Ibn Khaldūn's account is that, while there was sufficient economic means to afford parchment as the main writing surface, and there was not enough production of documents to outpace supply, this presented no problems to the scribal infrastructure of the state. However, once the production of documents began to outpace the supply of parchment, a new technology was needed. This, Ibn Khaldūn tells us, prompted Ibn Yaḥyā, vizier of a number of provinces of the Caliphate at the end of the 8th century, to suggest the manufacture of paper, which led to its use in both government and eventually in scholarship.

Ibn Khaldūn's story underlines the cheap cost of paper and the proliferation of written documents. More documents of all kinds were produced (from "[governmental] documents and diplomas" to full-fledged "books"), creating the need for a cheaper writing material. But this is only half of the story—with a cheaper writing material also came an increase in the production of written documents. This key idea has taken root in the historiography of both medieval Islamicate science and papermaking. Its earliest formulation comes from the historian Alfred von Kremer (1920, 58), who argued that "by the production of a cheap writing material, and its supply to markets both east and west, the Arabs made learning accessible to all". If a writing material becomes cheaper, then the books, and all the learning that depends on them, should become cheaper as well. Thus, a higher proportion of people will be able to acquire those books, and learning is rendered more "accessible to all".

How much more accessible? Gathering data from Ashtor (1969) and other sources, Shatzmiller (2024) provides the prices of many commodities—including books—across the Middle East, from about the 8th to even the 16th centuries in some cases. For the case of books, however, the region with the highest datapoints is Egypt, which modestly showcases the trend of cheapening manuscripts throughout the Middle Ages. (Figure 2) While the data by itself cannot confirm this trend, it provides a picture from the limited sample that has survived, with a rounded average of about 9 dinars in the early 11th century to about 1 dinar at the beginning of the 13th. This decrease, according to an analysis by Shatzmiller (2018), responds to commercialization of the

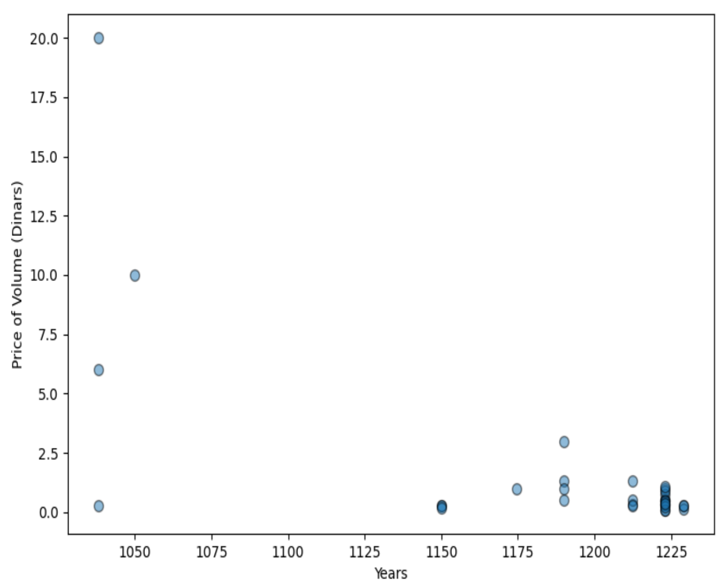

Figure 2. Scatterplot of Shatzmiller's (2024) data. The price per volume has been converted to dinars with a 1:40 dinar to dirham rate. (Goitein, 1965) Deeper blue represents a higher concentration of data points. Two extremely high early datapoints have been excluded from the data to enhance visualization.

textile industry, leading to a drop in the price of linen rags that ultimately provide the raw material for the production of paper, as well as the use of water power for cheap energy during the manufacturing process.

But the introduction of paper was not the only cause of the increase in book production. A crucial component is, of course, the center stage that the book takes in Islam. (Gruendler, 2020) Another reason is the economic boom that the Islamic world experienced as a result of the newly opened trade routes, spanning from the Atlantic to the Indian Oceans. Yet another is that the Islamic world had developed a sophisticated social technology for the copying of books, where multiple scribes would simultaneously copy the original text, dictated by the author themself. I am referring to the practice of the *imlāʾ* or dictation (see Behrens (2018) ch. 8 in the context of Mamluk Egypt and Syria). This has been argued for profusely by Bloom (2001, 2017), although Déroche (2007: 205) takes copying to be "in the majority of cases … a solitary experience". Regardless of the extent of the contribution of the *imlāʾ* to the overall increase in book production, however, the main point still stands: that the new writing material could not have been its only cause.

All of this suggests a very complex web of causal influences leading to the "learning boom" of the medieval Islamicate period. Nevertheless, the idea that paper was a key innovation for Arabic learning in general, and Arabic science in particular, is common across much of the literature on Islamic science and papermaking, as we'll see. The idea is reasonable enough, and many parallel developments buttress it. In architecture and engineering, for example, paper expanded the ability to draw sketches of a project. (Bloom, 2001; Grafton, 2001) The problem with the case of natural sciences, like astronomy, is that no equivalent story exists for just *how* the introduction of paper contributed to them. Part of the working assumptions seems to be that the science produced in the medieval Islamicate world is not just more numerous, but that it represented progress with respect to the available Greek, Persian, and Indian corpuses. But the idea that paper led to more books is not enough to explain how science improved with the help of paper. We seem to have both independently, but the jury is still out as to how they may be related.

This case of paper is but an example of broader questions about writing supports. We have seen how one of the central arguments regarding paper and learning is the cheapening of the costs associated with the production of texts. But, as is, this proposal is too vague: How, exactly, does a more affordable writing support condition scientific production? How we answer this kind of question will contribute greatly to the understanding that we have of scientific production more generally. We saw how cuneiform writing, for example, limited how efficiently the scripted information could travel. Similarly, how and when cheaper writing supports are introduced speak indirectly about how economic fluxes contribute to the constraints on information production and reproduction. These constraints are central if we want to understand the limitations and possibilities inherent in the materiality of scientific production.

To tackle these issues, I will consider how the spread and use of rag paper in the Iberian Peninsula might have contributed to the developments in astronomy that occurred from the 11th to the 13th centuries. Immediately before providing his explanation for the paper boom in early Islam, Ibn Khaldūn uses Iraq and al-Andalus as the two regions with the most concern with "scholarly writings and (official) records" (1967: 328). And indeed, the Iberian Peninsula was the first Western European territory to use paper as early as the end of the 10th century, and to transmit the technology to the Christian domains of the territory, ultimately to Italy in the mid 13th century, where industrial production of paper would take off. Explaining this correlation is the basic motivation for the hypothesis of this paper. Before we do that, however, I will consider the problem from a more abstract point of view. Rag paper was recognized as a momentous invention by many actors across the Medieval Islamic world, and the Iberian Peninsula introduced the material as a consequence of being interwoven in this huge polity. Why did the Islamic world value paper so much? How did those actors conceptualize their relationship to this writing material? These questions will help me form the basic framework I will use to then study the case of the Iberian Peninsula.

2. The hypothesis: Paper and the intermediary stages of scientific production

Paper arrived to the Islamicate world at some point of the 8th century AD. The story goes that, in the midst of the Arab expansion eastward, a battle was held at the banks of the Talas river between the ʿAbbasid general Ziyād ibn Sāliḥ, and the Tang general Gao Xianzhi. Ibn Sāliḥ and his men won the battle, after which he brought two captive Chinese papermakers back to his base in Samarqand—and with them, the technology of papermaking.[1] This story, however, is likely nothing more than a story, since there is no contemporary telling of the events, and we know that paper was already being used, and probably made, in Samarqand before the battle of Talas.[2] Whatever the case, we know that around 794 the caliph Hārūn al-Rashīd established paper as the official writing support for government documents. From there, the technology spread all across the Islamicate world, albeit more rapidly in the Eastern regions or *Mashriq*.

Before paper, however, the Arabs had a number of alternatives at their disposal. Ibn al-Nadīm (1970, 39) tells us in his *Fihrist* (ca. 987) about some of them. These include metal surfaces, used when writing was meant to last "for eternity", as well as wood, bark, the leaves of trees, silk, animal skins, and then finally papyrus and parchment. Because paper spreads westward from the ʿAbbasid centre, it coexisted with other writing surfaces—mostly papyrus and parchment—throughout the western territories at least until the 11th century.

[1] This is the account we find in Thaʿālibī's *Book of Curious and Entertaining Information* (1968).

[2] See Bloom (2001) and Rustow (2020) for a more contemporary viewpoint.

(Loveday, 2001, 21) This was especially the case in al-Andalus, the northwesternmost province of the Islamicate world, for reasons that we'll see.

Something noteworthy is that al-Nadīm chooses to frame his survey on writing supports from the point of view of the *function* they deploy, in particular its durability. Before the Flood, he says, "the peoples wrote on copper and stone for the sake of durability", and indeed metal surfaces are used when meant to last "for eternity". (1970, 39)[3] Later on, the emphasis shifts to the raw materials present in the different territories where the writing surface is manufactured. Egyptians write on "Egyptian paper made from the papyrus reed"; the Chinese on "paper made of ḥashīsḥ, which is the most important product of the land". Thus, the choice of writing support seems to depend at least on a dual condition: the availability of the raw materials, and the particular function each support is meant to fulfill.

But, of course, available raw materials and satisfaction of a function is not enough. Any writing surface will be generalized only when there is sufficient demand for it. That demand, however, can be stimulated or prevented artificially, for example from a political power. The initial thrust in the expansion of papermaking, as we've seen, came from its promotion in ʿAbbasid administration. Conversely, demand can also be blocked from within the administration: Both Frederik II and Alfonso X prevented the use of paper in their administration, to different degrees. Frederik II outright banned it, while Alfonso X relegated it to less important kinds of documents, as we'll have a chance to see.[4] Nevertheless, barring the influence that top-down interventions can have, one can still ask what the key factors are that make a writing support a more desirable alternative against the other available options. This goes back to the issue of trade-offs I mentioned in the introduction: What factors make a writing support a better option for the recording and processing of information?

From an abstract point of view, one wants a writing surface to optimize for at least the following characteristics. First of all, you want the surface to be durable. An example of an extremely durable support are al-Nadīm's metal surfaces, which retain what's written virtually "for eternity". Yet a slab of metal can be extremely heavy against alternatives like papyrus or parchment. Palm leaves, conversely, are more lightweight, but of course degrade more rapidly.[5] This suggests another desirable characteristic: the surface should be easily

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[3] Al-Nadim's concern for durability is also present in his recounting of Abū Maʿshar's story (1970, 576).

[4] For the case of Frederik II, see Powell (1971, Title LXXX). See García Díaz and Montalbán Jiménez (2005) for an interesting example of state intervention on paper use in Alfonso X's court.

[5] On top of this, palm leaves require a laborious writing process, where one first engraves the script with a stylus, and then fills the crevices with ink, washing off the ink left on the surface with water. For a nice tutorial on this often-overlooked method of manuscript production, see this video.

manipulable.[6] Manipulability involves both the extent to which supports can be moved through space, and the ease with which script can be laid over them. Heavy surfaces are difficult to carry around. Writing supports can also lack manipulability when they are too big, or too long. The classic example is the papyrus scroll, a standard surface in antiquity before the generalization of the codex. Surfaces that require carving, on the other hand, require a long time to lay down the script. In the previous footnote, we mentioned South Asian palm leaves, but a European example can be found in wax tablets during wintertime, which harden as a consequence of the cold temperatures. Finally—and importantly for our purposes—writing supports should be affordable, as Ibn Khaldūn stressed. Parchment, especially the more refined types like vellum, is lightweight, durable, and sizable to different uses, but it was among the most unaffordable options, if not the most unaffordable. This led to practices like abbreviating words in manuscripts to save writing space, or the production of palimpsests, losing the entire document one wanted to write over. (Cortés, 1988)

As I said earlier, what surface one decides to use will depend on the function that it's meant to serve. This assumes, of course, that there are viable alternatives at hand. But it is still an important point for my argument, because it relates the cost-benefit analysis to the effect the surface has on intellectual labor. But sometimes writing supports served other kinds of functions. Costly vellum would be employed, for example, to create illuminated codices or sacred books. Another factor that plays into the use of a writing material are the assumptions and beliefs associated with them. Much of the Christian world had strong negative perceptions of paper, which they associated with the "Saracens", or even the Jews. (See Section 4) Conversely, for the Islamicate world paper acquired a veneer of prestige that helped its generalization across literate circles. (Rustow, 2020) These perceptions often permeated into judgements about paper's durability and general quality.

Arab paper managed to nicely satisfy the requirements for durability and manipulability—much like papyrus, but mostly parchment, which was comparatively much more durable—without having to sacrifice affordability. While it is extremely difficult to get an accurate estimate of the prices of these materials for Medieval Islam (see Loveday, 2001, 23), it is nonetheless agreed that in the Islamicate world, where papermaking had achieved a "considerable degree of excellence" (in ibn Khaldūn's words), it soon became a much affordable alternative to papyrus and parchment. Papyrus was not only less durable than paper (as it easily unraveled with time), but much more expensive far from the Nile, where proximity to the raw materials made it more

[6] The *Book of Curious and Entertaining Information* also wrote about the manipulability of writing supports, in the context of Arab paper. Paper superseded papyrus in Egypt "because it looks better, is more supple, is more easily handled and is more convenient to write on" (1968, 140). So, while not the focus of my argument, considerations of the sensorial experience of writing—looks and suppleness—could also have played a part in the choice for paper. This would be along with a desire for durability, which is stressed both by al-Thaʾālibī and al-al-Nadīm, and manipulability: "more easily handled". And, finally, al-Thaʾālibī highlights the ease with which one can write on paper, being "more convenient to write on", perhaps in contrast to surfaces that require carving, like metal or palm leaves.

economically competitive. Parchment was even more expensive than papyrus, requiring a little above 200 sheep to manufacture a whole Bible, according to modern estimates (Kurlansky, 2016, 80). Paper, then, was a very light-weight material, and among the most durable ones, which could be manufactured at a very competitive price where papermaking industry was established.

Turn now to the implications that this could have had for medieval scientific production. We can pick up the thread we began at the introduction, and ask: How does a cheap writing support, like paper, change the conditions in which science is made? Note how the question is about the *conditions* of scientific production. Specifically, I want to argue that a cheaper writing support transformed the infrastructure involved in the intermediary stages of scientific practice, practices which crucially include the processing of information in the form of observational note-taking, or mathematical computation. But before we get into the details of this, let's address the main idea: How can paper affect the infrastructure of scientific production?

The idea that paper had at least some influence in medieval science is, as I already noted, conventional wisdom in the historiography of Islamic science. The idea is motivated by the great development of Arabic scientific culture, and the possibility of relating that development to what occurred to other disciplines with the advent of paper. Grafton (2021, 17) expresses this motivation very clearly:

> Muslims learned to compute, using Arabic numerals, with pen and ink on paper rather than an abacus or dust board. Cartographers who revived and expanded the techniques inherited from the Greek and Roman world made their maps on paper. Architects used paper to draw up formal plans for buildings. Paper, the nomadic medium, provided essential aid to societies that were remaking themselves in the pursuit of many forms of knowledge.

As we can see, the claim is about how paper aided the development of mathematics, and the pursuit of knowledge more generally. An interesting aspect of Grafton's argument is the analogical argument used with cartography and architecture. This is a parallel that is widely used in the literature (see also Latour (1986) and Bloom (2001) for other examples). The underlying idea is reasonable enough. In order to arrive at a final product, be it a mathematical treatise, a map, or a final building, it is required to go through a number of steps in between—in the mathematical case, one needs to work out (say) how a particular geometric construction is equivalent to another; or, in the architectural case, the more frequent that one can try out the design of a project means, up to a point, that the final result will improve. Arguments like these suggest that the use of paper, like in other practices, must have revolutionized science.

This reasoning, however, is never explicitly presented. Usually, as in the case of Grafton, the point remains more circumstantial: Paper is observed to have helped many practices in the medieval Islamic world, and we can see it present in scientific practice, so it stands to reason that it must have been helpful there, too.

However, the details matter. How exactly did paper transform scientific practice? The hypothesis I want to put forward acknowledges the assumption (expressed in Grafton) that is implicit in the literature, but aims at making it concrete: Paper changed the material conditions implied in these intermediate processes that are required before the final product is achieved—by making architectural and cartographic sketches, or (as we'll see later) mathematical scratch work, observational note-taking, and collaborative work, more accessible. This implies an off-loading of cognitive resources into material resources, ultimately improving the general ability to work through problems, both individually but also as a community of practitioners, that is common to all these practices.

To appreciate this point, it is important to understand the contrast between paper and its alternatives in intellectual labor. Parchment and papyrus, while manipulable and durable (especially parchment), required the laborer to assume a great economic cost. In most of the labor associated with mathematical practice, the overwhelming majority of paper used does not serve any purpose by itself: Once used, and especially once a solution is arrived at, the documents lose their purpose. If your material is expensive, the cost of opportunity skyrockets. This characteristic of mathematical production is crucial to understand astronomical practice in the Middle Ages, both theoretical and computational.

An interesting contrast is presented with wax or dust tablets, a writing support which remained in wide use even all the way through the 19th century in Europe. (Lalou, 1993, 280) These tablets consisted, at least in their standard form, of a wooden (sometimes metallic) frame, containing a layer of bees wax or sand, the latter being more prevalent across the Islamicate world. The user would write over them with a simple stylus and then flatten them (*tabula rasa*) so as to have a new clear surface. These were very commonly used in teaching contexts, as well as for drafts of work that would be eventually copied onto a more permanent material. (Clemens and Graham, 2008) Because of these benefits, human ingenuity produced many different versions of this technology, like multiple tablets joined together that increased the available surface while allowing for easier portability. (Figure 3)

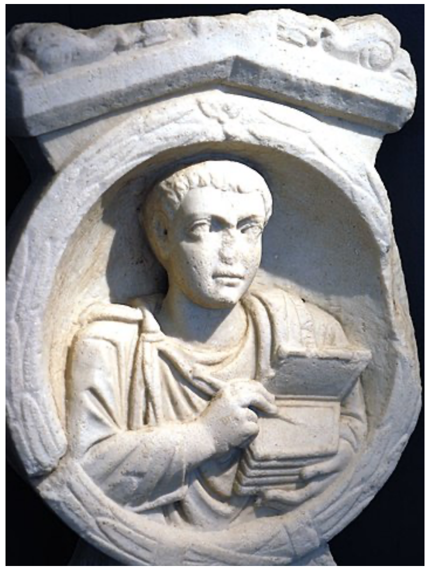

Figure 3. Roman tomb relief of a scribe at Flavia Solva, modern Styria. The scribe is holding a set of wax tablets while holding a stylus. (Wikipedia Commons)

These tablets had the great advantage of allowing for the writing of drafts, and in general work that was not meant to last. Why, then, would paper be used for this purpose, rather than sticking to the tablets? Because of their wooden framing, there is a limit to how the size of the tablets can grow, without becoming "non-manipulable". As a result, there is a limit to how much scratch work it was possible to do on them, before you're forced to erase the previous work. Paper does not present the same restrictions, and not only can be scaled in size much better, but allows for many more units of it. This results in an overall much more convenient material, which could be written on without having to erase the previous work. The importance of this fact is reinforced as we begin to think of complex tasks, like the design of a building, or working through a laborious mathematical computation. In these and other cases, one needs not just a material on which to write scratch work and to lay down a mental process—it is just as important to have that work available, so one can come back to it and build up on what's already been externalized from the mental process. As a result, paper also becomes more convenient for the storing of information, since having an inexpensive, relatively lower volume, repository of information allows one to keep just more quantities of it.

So, summarizing what we have, we could put the hypothesis like this: Paper represented, against the available alternatives, a more affordable option for the off-loading of much of the cognitive work associated with scientific production. In particular, it afforded a viable surface in which to lay down the intermediary stages before arriving at the final product. Other writing supports, like parchment or papyrus, were too expensive for this kind of use, and alternatives like wax tablets and dust boards didn't scale appropriately for some of the more complex tasks involved in technological or scientific practice. In this sense, paper struck a nice balance between the material's durability, manipulability, and affordability. The balance was not perfect (paper wasn't exactly cheap, and it wasn't always of the greatest quality), but—and this is the point—it was better than the alternatives, all things considered.

While this cannot be the whole explanation for the expansion of paper across the medieval Islamicate world, it should serve (or so I hope) to identify what aspects of the writing material made it a competitive alternative in the context of scientific, and also technological, production. But the argument is still too abstract. In the next section, I will zoom in on the Iberian Peninsula, which introduced papermaking way after its generalization across the Middle East by at least a century. As I will try to show, these ideas will carry over to the astronomical practice of the region, highlighting more general aspects of scientific practice.

3. Paper in Andalusi science

3.1 The introduction of paper in the peninsula

The period that goes from the 11th to the 13th centuries is a moment of intense scientific activity in the peninsula. Right after the fall of the Caliphate of Cordoba (929-1031), the peninsula had already spent a few centuries absorbing the thriving culture of Baghdad, and acquired the resources to begin contributing on its own to the burgeoning Arabic scientific tradition. In this period, al-Andalus is able to house both developers of Ptolemaic astronomy like Maslama al-Majrīṭī (d. 1007), and al-Zarqālī (d. 1100), whom we'll talk about later, as well as anti-Ptolemaic philosophers like ibn Rushd, or Averroes (d. 1198). (Sabra, 1984) Finally, as the Christian kingdoms gained the upper hand and progressed in their southward expansion, scriptoriums like that of Alfonso X (d. 1284) produced a number of important works in astronomy and astrology, including the tables bearing his name. Meanwhile, a process of translation of Arabic scientific culture, which began in 10th century Catalunya (Zuccato, 2005) and culminated in 12th to 13th-century Toledo, was transmitting works like Ptolemy's *Almagest* or Euclid's *Elements*, along with many other Arabic scientific works.

At the same time, paper was gradually being introduced, and used, across the peninsula. The first paper record that we have in the peninsula is the Missal of Silos (Silos Monastery, Ms. 6º Breviarium et Missale Mozarabicum), a missal detailing the Mozarabic rite composed of 157 pages, 38 in paper, the other 119 in parchment. While the exact date of its creation is unknown, most estimates situate it somewhere between the last quarter of the 10th century and the early 11th. (Valls i Subirà, 1978: 102) According to Gayoso Carreira (1994), this missal might have been manufactured in Nájera, then Christian Kingdom of Navarra, which would indicate that at least by the end of the 11th century, Muslims were not the only ones who made use of paper.[7] Papermaking reached its peak of sophistication in the peninsula around the 12th century, when we have the first mention of a paper factory in the city of Xàtiva.[8] In 1154, al-Idrīsī wrote about the city: "Paper is there manufactured, such as cannot be found elsewhere in the civilized world, and is sent to the East and the West".[9] But Xàtivan paper was a luxury product, and it is likely that for domestic consumption paper from less reputable mills was in demand. By approximately 1254, however, we find the first documents speaking of a paper factory in Fabriano. This new factory kickstarts the Italian predominance in the paper industry and marks the end of the first period of papermaking in Europe.

It is also worth stressing, however, that paper never was the exclusive writing support throughout the Medieval period. The peninsula remained a center for the manufacture of leather, with "cordovan leather" usually made from goatskin. (Vallvé Bermejo, 1980; Constable, 1994) This probably contributed to the prevalence of parchment, a feature that it shared with the broader Maghreb. The Calendar of Cordoba, for instance, states that "We make parchment from fawn and gazelle until the end of July", with the skin of wild animals having a reputation for being better quality. (Pellat, 1961, 90-1) In this sense, paper does not substitute other writing supports like parchment or wax tablets, but rather is *added* to the existing alternatives. This makes the peninsula a particularly interesting location for the study of the function of each material in the process of knowledge production, highlighting a question of function: Why paper, as opposed to the available alternatives? In the rest of this section I will try to shed some light on the issue, looking at astronomical tabulation as an instance of medieval scientific practice.

3.2 Communication and collaboration in the Toledan school

The tradition of planetary and astral tabulation started with Ptolemy's *Syntaxis*, but it was only within Arabic science that the practice took off and became one of the central features of astronomical and navigational practice. However, having lost most of Ptolemy's work, and not having access to the recent developments in Arabic astronomy, Christian astronomy was mostly focused on star classification and

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[7] Bongianino (2022, 79n53), however, situates its manufacture in Toledo.

[8] Note that Xàtiva's is likely not the first place to produce paper. Valls i Subirà (1978, 89), as well as Escolar (1993, 127), argue for the presence of paper mills in Córdoba already by the 10th century. The crucial difference in Xátiva (Burns (1981, 1985) argues) is the introduction of plants with a labor base that extended beyond the family unit, and which incorporated waterwheels starting form the 13th century.

[9] Translation by Hunter (1978: 473). I have not been able to find the original source for this often quoted text—all of them remit to Hunter 1978. For paper trade in al-Andalus, see Constable (1994: 191 ff.) and Burns (1985: 173-176). Burns provides an approximation to paper pricing in the Kingdom of Aragon towards the second half of the thirteenth century, mostly through its associated tariffs. These fluctuated enormously, ranging from 2 sous per box (which contained 1300 sheets of paper) to 20. The problem here is that these tariffs are more indicative of the governing body's economic policy, rather than the cost of the commodity itself.

calendrical calculation (or *computus*).[10] Thus, when Adelard of Bath translated Maslama's adaptation of al-Khwārizmī's tables to Latin (possibly in 1115 – see Haskins (1911)) and gifted it to the then Prince Henry of England (d. 1135 AD), this was the first time that Northern Europe had had access to a work of this kind in a long time. Medieval astronomical tables were ultimately a practical tool to aid in the calculations of the positions of the planets and the stars, as well as other celestial phenomena like eclipses or constellations. In the Islamic tabular tradition (what are known as *zījes*), this type of document offered a host of other uses. While it is not clear what the main purpose of a *zīj* was (King et al., 2001), it is clear that they served many purposes and were widely used among all kinds of professions. A geographer could, for instance, use the mathematical methods of the *zīj* to calculate the *qibla* in their locality; or an astrologer could set up a horoscope with them.

Although, starting from the 10th century, different traditions of table-making begin to emerge, there are common elements to all *zījes*. Every *zīj* would begin by dealing with calendrical problems, like defining the eras or defining the *madkhal*. In addition, they would include chapters containing a table of trigonometric functions (crucial for the computation of the *qibla*), as well as chapter dealing with astronomical time keeping, which were based on the mathematics of spherical astronomy. Nevertheless, the main components of the *zījes* were the tabulation of the mean motions of the planets, which needed correction by equations to obtain the actual ecliptic motions. These would be followed by tables relating the direct and retrograde motions of the planets, as well as solar and lunar phenomena. Finally, they would also include tables for geographical and astrological purposes, as previously stated. Added to these contents, one would have to include special tables that pertain to particular tabular traditions. In al-Andalus and the Maghreb, for example, one could find tables relating to the phenomenon of trepidation, which postulated an oscillatory motion of the equinoxes. (See Samsó, 1994)

That was the case also for the Toledan tables—a set of tables whose production begins around 1061 and which remained the main reference in Europe until their replacement by the Alfonsine tables in 1256. Toomer (1968) gives a list of the main phenomena that are tabulated: Calendrical equivalences (Tables 1-11), trigonometric and spherical astronomy (12-27), mean motion of the planets (28-36), solar, lunar and planetary equations (37-44), planetary latitudes (45-48), planetary visibility and retrogradation (49-51), eclipses (52-80), miscellaneous (81),[11] star catalogue (82-82a), geographical coordinates (83), and finally astrological tables (84-85). This should help us get an idea of the wide variety of contents that the document must have had. Unfortunately, the original Toledan tables are not extant.

[10] Which, of course, has a sophistication and interest of its own. See McCluskey (2000) and Falk (2020).

[11] This is a table for precession calculations ('motion of the eighth sphere') based on the trepidation theory attributed to Thābit ibn Qurra. (Toomer, 1968: 8)

A notable feature of the Toledan tables is that the process of elaboration was not due to a single astronomer, but many, led by *qāḍī* Ṣāʿid al-Andalusī (d. 1070) with al-Zarqālī (d. 1100) as its main contributor (Richter-Bernburg, 1987). Due to the specifics of the process of reception of Arabic science into the peninsula (see Samsó, 1992), this collective had a number of distinguishable characteristics, constituting something like a "Toledan school" of quantitative astronomy. Isaac Israeli ben Joseph, Toledan astronomer who in 1310 composed a survey of the astronomy of his time, wrote of a group formed around 1040, who

> lived in the city of Toledo and other lands of Spain, about a dozen, including some Jews. … These devoted themselves to studying and experimenting through the observation of the heavenly "signs", and they would not desist until having progressed in this science and elucidating many of its principles. The aforementioned Ibn Ṣāʿid … loved science and those that devoted themselves to it … and he offered what he possessed to his collaborators, providing for them and giving them stipends. … These research colleagues installed in the city of Toledo various precision instruments to observe and regulate everything necessary for the verification both of the position of the planets and their positions in the Zodiac, as well as their speed in each era and the duration of the solar year and the lunar month.[12]

If we assume what Isaac ben Joseph is saying, we should imagine a group of a "dozen" individuals, devoted to observational astronomy. We know, of course, that this group also worked on quantitative astronomy, which can be attested not only by al-Zarqālī's work (see Millás Vallicrosa (1943-1950)) but Ṣāʿid's own *Correction on the Movement of the Stars*, which is not extant. However, Ṣāʿid's Ṭ*abaqāt* (Llavero Ruiz, 2000) allows us to get a better picture of the crucial features of the school's particular astronomical features. According to his account, 11th-century Toledo still made use of the computational methods from the *Sindhind* (which by then had progressively been abandoned in the *Mashriq* in favor of Greek ones), due to the influence of Maslama's adaptation of the tables of al-Khwārizmī. Ṣāʿid considers Maslama the beginning of Andalusi scientific achievement (169[13]), and indeed he traces to him the Toledan tradition of astronomy (Figure 4) On top of this, the broader peninsula

[12] *Yesod Olam*, book IV, ch. 7. Translation from a Spanish version in Llavero Ruiz (2000, 14).

[13] Pagination from the Arabic original.

became one of the strongholds of trepidation theory—also progressively abandoned in the M*ashriq*—mostly through al-Zarqālī's trepidation model, itself influenced by Thābit ibn Qurra's. (Goldstein, 1965)

The question that interests us here, of course, is how this Toledan school worked, in the precise sense of what practices characterized their process of scientific production. Astronomical tables, as a subtype of

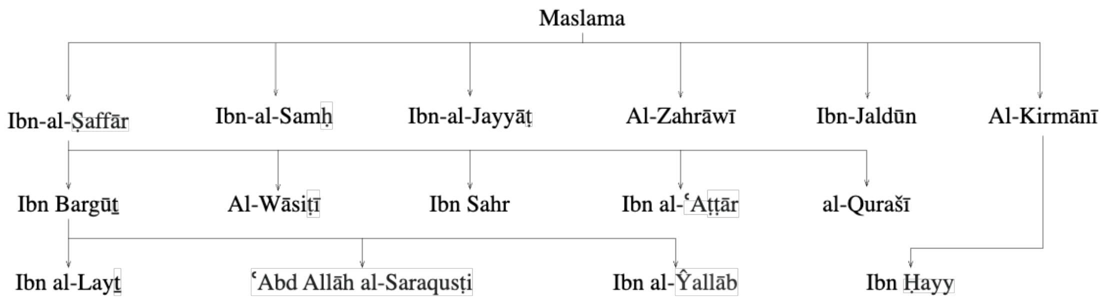

Figure 4. "Genealogical tree" of the Toledan school, as reported by Ṣāʿid al-Andalusī in the Ṭ*abaqāt*. Adapted from Verner and Samsó (1981). Ṣāʿid excludes from this tree both himself and al-Zarqālī. On the relationship between those two, see Richter-Bernburg (1987).

intellectual labor, depend crucially on a workflow of writing that articulates all other scientific practices, from observation to computation and theory formation. Observations have to be recorded, theories are communicated, and computations—as we'll later have a chance to explore—depend on writing to effectively exercise their function. So, what workflow characterized the Toledan school, and by extension the Toledan tables? In the prologue to his *Fixed Stars*, (ca. 1084-1085) al-Zarqālī describes briefly what the process of elaboration consisted in: "We verified constantly and attentively the observations for the sun, the moon and the stars that we were capable of obtaining, relying on trustworthy individuals, for the span of 25 years. After that I began to write the *Comprehensive Work on the Solar Year*, such that with it I was able to confirm everything we were capable of."[14]

This text, I think, suggests a number of questions. First, al-Zarqālī talks about verifying observations. Here, he is presumably talking about the process of checking the values of the tables by al-Khwārizmī and al-Battani, as well as others', perhaps by making the observations themselves, and adapting them to the meridian of Toledo. How were these observations carried out? What kind of instruments were used in the process? And, most importantly for my purposes, how were these observations written down? But, not only that, he's also saying that these observations were carried out for 25 years and in collaboration with other individuals. How

[14] Translation from Millás Vallicrosa, J. M. (1943-1950). *Estudios sobre Azarquiel*, with Arabic original and Spanish translation, in p. 279.

did the communication between these individuals occur? How were the observations stored until they were needed, either to move them around or to retrieve them?

Notice how writing supports present here a particularly economical lens through which to understand these questions. Since so much of the workflow streams through writing supports, we can not only get a view into the work of an individual, but also ask questions about collective aspects of scientific production. As we've just seen, the Toledan tables were a process that mobilized multiple individuals, and in this case 25 years of observations. This makes sense if we bear in mind the amount of information that these tables include. All that information had to be first of all recorded, but also stored, at least for 25 years, in a way that allowed for transportation and communication. Here the analysis should compare the available alternatives in the period––wax tablets or dust boards, parchment, and paper.

While we don't know the details of dust board and wax tablet use in the peninsula during the 11th and 13th centuries, it is safe to assume that both writing systems were used simultaneously. Thus, it is likely that Christians, both Andalusi and northern, followed this trend and made use of the technology. The contemporary literature can be somewhat confusing, since the word for the dust board (ḥ*isab al-ghubār*) is the same as that for the Hindu-Arabic numerals. This is because the dust board was a system that the Islamic world inherited from the Indian east along with the numerals. (Kheirandish and Berggren, 2014) In fact, initially they were one and the same thing, and only after al-Uqlīdisī (Saidan, 2012) and other practitioners do they start to come apart. The Ṭ*abaqāt* also speaks of ḥ*isab al-ghubār*, as "the simplest, most concise, accessible, of most original structure and easiest to learn system" for arithmetic operations. (58) However, the sentence makes it clear that Ṣāʿid is referring to the numeric, rather than the writing, system, and by extension arithmetic at large. This use of the word is retained in the Arabic-speaking part of the peninsula at least all the way to the fifteenth century—Al-Qalaṣādī (d. 1486) wrote a pedagogical text titled *Unfolding the secrets of the use of dust letters* [ḥ*isab al-ghubār*], a treatise on arithmetic. But, as we know, paper was already being used and produced in the peninsula in this time, so the question comes down to one of function, rather than which of the two dominated in any absolute sense.

In terms of function, then, wax and dust tablets are instruments of limited size, and so present the problem that they limit the amount of observations (and computations) that can be recorded before they have to be deleted. This requires the use of a multitude of tablets, and as a result, they rapidly become unmanageable as they grow in number. Since both wax tablets and dust boards require heavy materials for their manufacture (like wood or minerals), and that makes transportation—and hence the transmission of its contents—difficult. In addition, 25 years of observations require the implementation of some form of storage system. But for this, dust boards couldn't be used, since they were limited to quick uses and the accumulation of multiple dust boards for the storage of information is unheard of, at least from what I've been able to find.[15] Paper—already in wide use and likely locally manufactured—was a support that allowed for storage and much easier transportation, something required for a group of individuals that accumulated observations for long spans of time, as al-Zarqālī recounts.

Of course, the use of parchment would have easily solved these problems, as it is just as manipulable as paper, and in general much more durable, making it a better candidate for the storage of information. But it is important to remember that, even if al-Andalus remained a center of parchment-making throughout the Middle Ages, the uses of parchment were still limited to formal documents, due to the cost of the material and the aura of sanctity that surrounded it. While it is (as mentioned earlier) very difficult to get an exact idea of the cost of these writing materials, (see Loveday (2001), 23) there is no reason to suppose that they followed very different trends in al-Andalus than other regions of the Islamic world, especially given the development of the papermaking industry in the peninsula. In this way, paper was not just a manipulable, lightweight alternative that allowed for storage of information for years at a time (parchment was already that), it was crucially a relatively inexpensive material to do so.

Before we jump to the Alfonsine tables, it's worth to emphasize that this discussion of the Toledan tables has to be put in the context of the broader Toledan school, which used observations and methods for the production of tables, but also other products, like al-Zarqālī's treatise on the *Fixed Stars*, which explicitly alludes to these observations. Work like Toomer's (1968) has questioned the originality of the Toledan tables, suggesting that many of differences with respect to al-Khwārizmī's and al-Battāni's may be due to copying errors, with perhaps the only the mean motion tables (28-36) being an original contribution from the Toledan school. Regardless of this particular issue, I think there is sufficient reason to take the Toledan school to work collectively on a manifold of projects, of which the tables were just a part, and hence similarly influenced by the school's practices. The point here is, then, not just that paper transformed the material conditions of astronomical tabulation, but also the material conditions of the broader structure of practices that source the production of those tables. This point will be present again in the next example.

3.3 Computational labor in the Alfonsine tables

[15] Added to this, there are considerations about the writing tool, more specifically the performance of a stylus versus a pen. One can wonder, for instance, the speed with which one can write down observations in different writing materials, and how that might have affected the observational practices. Famously, wax hardens in the winter months, which made writing a slower and more arduous process. But in this paper I'll restrict my attention to writing supports.

The Toledan tables were quickly translated and adapted to other regions of the European continent, and by the 12th century they had already been adapted to the Christian calendar, in tables like those of Toulouse or Marseille (Chabás and Goldstein, 2012). However by the 1260s, king of Castile Alfonso X was assembling a group of astronomers for the purpose of adapting and correcting the Toledan tables, a work they completed by 1272, named *Libro de las tablas alfonsíes* (Book of the Alfonsine tables). We have some insight into the process of elaboration of these tables thanks to the book's *Canones*, written by the two leading members of Alfonso's astronomical team: Yehuda ben Moshe ha-Kohen (d. late 12th c.) and Isaac ibn Sid (*ibid*).

These court astrologers open up the *Canones* emphasizing the collaborative character of the astronomical enterprise: "the science of astrology cannot be figured out but through observation, and the observations that characterize the wise men that are successful in this cannot be completed in the life of a single man. Indeed, when this is achieved, it is so through the action of many men, one after another, in long stretches of time." [2][16] This shows how the practitioners surrounding Alfonso's *scriptorium* not only were themselves the product of a tradition (on this, see Samsó, 2020), but also perceived themselves as continuous with it. Al-Zarqālī (or *Asarquiel*) is the very first name to appear only after the Cesar's, and indeed al-Zarqālī's tabular production forms the core of the inherited corpus that Yehuda and Isaac decide to amend [*endereçar*].

Much like in the case of the Toledan school, questions about the process of production throw us into questions about material production. What can we gather about the writing process that characterized the team's production? While not much is known about the specifics of how the Tables themselves were produced, one may make some inferences on the basis of more general features of the Alfonsine "workshop", in Fernández Fernández's words (2019). The *scriptorium* of Alfonso X, for example, frequently used paper as material for legal documents. We know this, in fact, because we know of other documents which ask for the renovation of paper documents into parchment.[17] But the very regulations of the *scriptorium* suggest that this was the case for all types of documents, namely that paper remained in wide use for drafts and documents considered less important. Fernández Fernández has pointed this out in relation to the Cantigas de Santa Maria (2012, 2012-2013), but the generality with which those regulations are formulated suggest that this applied across the board, including in the extensive scientific activity of the Alfonsine *scriptorium* (Fernández Fernández, personal communication). It is worth highlighting, in this context, two manuscripts of scientific content that were copied on paper in the *scriptorium*: the Ms. Or. 152 in the Laurentian Library (Florence), a collection of scientific treatises

[16] Bracketed numbers indicate sentence number from the MS 3306 in Biblioteca Nacional (Madrid), as per Chabás and Goldstein's (2003) edition.

[17] The Archive of the Cathedral of Seville (Box 18, nº 1) keeps a letter from Bishop Remondo petitioning (*petitio*) to king Alfonso X that he resends a letter, which he had previously sent in paper, in parchment (*cuero*). See López Gutiérrez (1992).

in Arabic with notes in Aljamiado by Isaac ben Sid, and the Ms. 915 in the Royal Library of the Escorial (Madrid), a copy of the *Almagest.* (See Villuendas (1978) and Comes (1990))

And so again, the issue at hand is not so much whether paper was used in general, or at all, but how the material was used—what *function* paper had within Alfonso's *scriptorium*, and more in particular in the astronomical production that came out of his court. As I just mentioned, the *Canones* begins [6] acknowledging al-Zarqālī's contribution to the Toledan tables, while asserting that his observations presented "inconsistencies" [10] that prompted Alfonso to command the reconstruction of many instruments to repeat some of those observations. Many of those observations are listed [11-15]. And yet the *Canones* suggest a more thorough process than mere observational labor. The very word used to refer to the observations [*rectificamiento*] also means to recompute, or even to reconsider. (Chabás and Goldstein, 2003, 126) Thus, one is led to think that the correction of al-Zarqālī's work goes beyond the observational, to extend into recomputing some of the previous calculations. Since the original Castilian tables are not extant it's not possible to ascertain their exact contents, but the *Canones* include tables that are of pure computational content, like a set of trigonometric tables (Chapter 37) which include a table of chords [*querda* or *corda*] computed in the Ptolemaic way (see Chabás and Goldstein (2003, 200-203) and references therein), as well as a table of sines [*signo*], of cosines [*signo del complimiento*], and versines [*saeta*].

In paragraph [51] of Chapter 37, the text describes how to compute the versine in the following way:

> If you want to obtain the versine, find the arc whose versine you want to know, and whose full chord and sine you also know, and multiply that full chord by itself, and name the result the square of the chord. And multiply the sine by itself and name the result the square of the sine. And subtract the square of the sine from the square of the chord and take the square root of the rest and that will be the versine of the arch you obtained.

That is, to find the versine of an arbitrary angle α, first we find its arc: the length of a chord subtended around an angle α of a 60-radius circle. The method described assumes we also know the sine. Paragraph [45] provides the expression for the sine as $\sin\alpha = \mathrm{crd}\ (2\alpha)/2$. Thus, knowing the chord allows one to find the sine. Then, we're told, square the sine and the chord, subtracting the first from the second and taking the square root of the result. We get the final formula

$$\mathrm{versin}\ \alpha = \sqrt{(\mathrm{crd}\ \alpha)^2 - (\sin\alpha)^2}$$

to compute the versine as a function of the chord and the sine. We are told in [52] that this is the method used to produce the table of versines.

Given that the original Castilian tables are not extant, it's not possible to get an exact figure on the number of entries that this versine table—or any other of the trigonometric tables—might have had. However, it is well known that any of these tables required an inordinate amount of computational labor. A look at al-Zarqālī's *Almanac* (Millás Vallicrosa, 1943-1950, 229) and al-Kammād's *zīj* (Madrid, Biblioteca Nacional, Ms. 4238) suggests that just the table of versines should have included within the order of 120 entries, each provided to three significant places, if we are to extrapolate from the table of chords (Chabás and Goldstein, 2003, 202).

How did this computational process take place?[18] While, to my knowledge, there is no literature that compares the evolution of computational techniques in the peninsula, Berggren (1986) has shown the evolution of computational techniques in the Islamic Middle East. Take the case of computing a square root, which as we just saw was needed for the computation of a table of versines. From al-Uqlīdisī's time to al-Kāshī's, as dust boards begin to slowly be replaced by paper, all the intermediate steps involved in the computational process are off-loaded, from the computer's working memory to the paper's writing space. The introduction of a cheap writing support makes writing space lose its value, and there is no longer need for the computer to retain all the computation in their memory. (Figure 5)

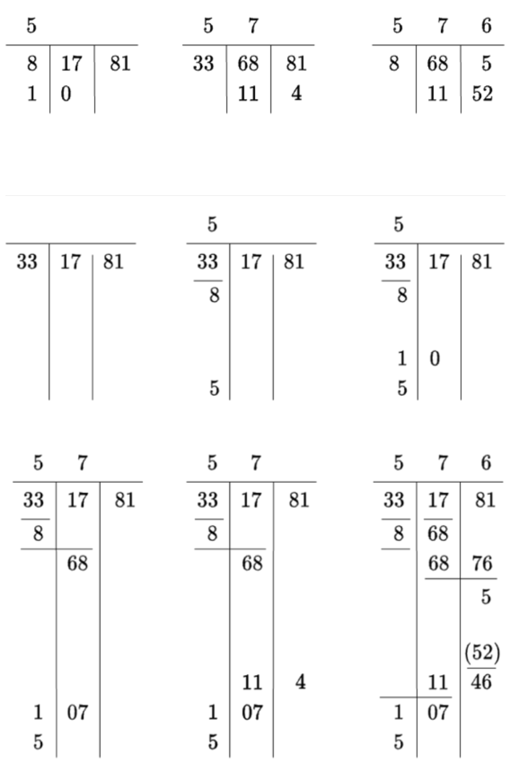

Figure 5. Evolution of the square root computation from al-Uqlīdisī (top) to al-Kāshī (bottom). Adapted from Berggren (1986). In both cases, the method is the same. To better follow the method, consider al-Kāshī's version. First, divide the number 331,781 into digit pairs from right to left (these are called "cycles"). The largest n for which $n^2 \leq 33$ is 5. Then subtract its square from the first cycle, getting 8. Double the root (5) inching to the next cycle, and find an x such that $(100 + x)x \leq 817$. This is 7, which is the next root digit. Subtracting $(100 + 7) \cdot 7 = 749$ from 817 gives us the remainder 68. Repeating this for the final cycle, $57 \times 2 = 114$, and finding x such that $(1140 + x)x \leq 6881$ gives $x = 6$. One can find decimals by iterating this process, finding $\sqrt{331{,}781} \approx 576.00434$. To compare: These computations, on dust board, had to be reduced to as much as a dust board allowed. One can imagine why al-Uqlīdisī might have so vigorously proposed switching to pen and paper, other than to maintain the reputation of mathematicians. (See Saidan, 1978)

[18] Here it would be impossible to do full justice to the complexity of computational methods that existed in the period, which combined both techniques with writing support as well as without it, including finger reckoning, the abacus, or even the use of multiplication tables (King, 1974). This exposition will thus only highlight features of a component in the rich life of medieval computational methods: pen and paper calculations.

What material technologies supported the computational techniques laid out in the *Canones*—and, specifically, what writing supports aided each part of the process? While, by al-Kāshī's times, paper had already been completely generalized and even papyrus had virtually disappeared in the *Mashriq* (Loveday, 2001: 21), parchment, wax tablets, and presumably dust boards were still in use in the peninsula. But the alternative of paper, in this context, offered an inexpensive writing support which still offered the light-weight, manipulable quality of parchment. We know that the *scriptorium* used paper in some form, and it requires no extra assumptions to extend this to its astronomical and astrological output. Parchment, of course, would have still been used for official documents (though not all of them, as we saw) and final manuscripts.

What about tablets and boards? Limited in the area that one was able to use before they become extremely heavy-weight and hardly portable in large quantities, they could have hardly competed in the realm of computational labor. If we think about mathematics (hence computational techniques) as a technology of script, we can see how any form of mathematical labor is materially dependent on its writing support—hence also in the kind of support. This becomes particularly poignant in cases of laborious computations, which require several intermediary steps before arriving at the final result. The more complex the computational process, the more that the intermediary steps need to be off-loaded to a writing support. Another way of arguing for this point is through the cognitive limits inherent to working memory. Working through a complex mathematical problem (or really any problem in the short term) requires the use of our working memory.[19] That is: one needs to retain in their mental workspace the relevant information for the given cognitive task, as they're processing the solution. One of the crucial advantages of mathematics as a technology of script is that it off-loads that relevant information, allowing for some independence from the hard cognitive limit to the working memory that we have as individuals. In this context, the technology of paper is a continuation of this trend brought by script: It allows for the off-loading of a higher proportion of the information required to process a complex mathematical problem, of which computations like the trigonometric tables are an example.

Hence, the improved infrastructure that paper brought with it implied, in the case of astronomical tabulation, a certain evolution in how data could be processed, stored, and communicated. With a new inexpensive writing material, which emulated the functional characteristics of parchment (it was light-weight, custom-sized, and considerably durable), the cognitive off-loading afforded by the writing surface was significantly transformed. Paper, thus, highlights the importance of the material infrastructure involved in the scientific process. Much like in the case of cuneiform script, the material conditions of information processing greatly conditions how that information is processed.

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[19] This said, the details of this relationship are still being worked out, with many existing hypotheses. For a review, see Raghubar et al. (2010).

4. A curse of intermediary documents in medieval scientific production

The last two sections conform the key idea that I wanted to propose: That the introduction of paper, *qua* writing support, transformed the conditions of scientific production—particularly astronomical tabulation. But this idea rests on a more specific hypothesis about the function of paper in medieval (Islamicate) science, as an intermediary document used for communicational and computational purposes. While the array of uses that are comprehended in this category is itself huge, there is nevertheless a kind of prediction associated with the concept: Intermediary documents are destined to be discarded, sold,[20] or reused, but not preserved.

This poses a challenge if we want to find documentary evidence of things like mathematical scratch work or communication between practitioners. A preliminary look at the archives suggests a rather bleak picture, since the overwhelming majority of scientific documents from the Iberian Middle Ages in other European libraries are written on parchment, with only a small minority on paper. There are two reasons, however, why we should not take this preliminary suggestion at face value. First of all, there is a certain bias in the kind of documents that libraries and museums usually hold. In the case of medieval manuscripts, luxurious, richly illuminated manuscripts are considerably overrepresented in their collections. This makes it more unlikely that we'd have access to documents on paper, especially given the prevalent opinion at the time among Christians against paper.

A couple of examples can illustrate this point. It is well-known how medieval Christians had somewhat of a pre-made mind regarding paper. One of the most famous renditions of this opinion comes from Peter the Venerable (d. 1156), who imagines what kind of writing surface God would use: "God, it says, reads the book of Talmud in Heaven. But what kind of book? Is it the kind we have in daily use, made from skins of rams, goats, or calves? Or, is it made from reeds and rushes out of Eastern swamps, or from old rags (*ex rasuris panorum*), or from some other vile material … ?".[21] Peter the Venerable was likely referring to the paper mills he had seen in al-Andalus, which he had recently visited in order to get a translation of the Quran. (Valls I Subirà, 1978) He was voicing a widespread opinion, which associated paper with Saracens and Jews, and considered it a low-grade material, at best a substitute for parchment. (Clemens and Graham, 2008) This was the same for Alfonso X, who as we saw relegated the use of paper (*pergamino de paper*) to non-official documents, as he considered it less durable, and generally less trustworthy.

[20] The reselling of writing material was common practice at least in the medieval Islamic world (Shatzmiller, 2018; Behrens-Abouseif, 2018). King (2007) recounts the case of the prolific astronomer ibn Yūnus (d. 1009), whose son sold all of his extant papers in the soap market.

[21] Peter the Venerable, *Tractates adverts Judaeorum inveterate duritiem*. Quote and translation from Blum (1934).

Alfonso X's policy, however, was in a way simply reflecting a practice that was already prevalent in the Castile of his time. Paper was considered a low-grade, disposable material, destined to be discarded once its purpose was accomplished. This highlights the segmentation of functions that different writing materials have: Parchment was designed for luxurious, final products; paper for intermediary ones. The problem here is that, as a result, the records one would usually use for historical study are lost (or at least most of them). Valls i Subirà (1978: 102) laments this very fact while recounting his own experience in the historical study of paper: "It is remarkable for the researcher", he says, "the existing lack of proportion between documents written on parchment, and those written on paper. … It seems like documents were kept strictly by the individuals. Only thus is it possible to explain the great quantity of foldings that each piece of paper has, as well as the stains brought about by the handling and the sweat. … In Castile, the use of paper was limited to documents of the lowest category. That led almost directly to their destruction, and hence the disproportion between parchment and paper."

While Valls i Subirà is not thinking of documents involved in the scientific process, the same reasoning applies. The regulations of the Wise King's *scriptorium*, as well as the extant scientific documents on paper that came out of the workshop, testify to a use of paper in line with the period, where the majority of final documents—particularly those considered valuable—required transcription onto parchment. In this context one can understand, perhaps, why Bishop Remondo may have asked for his *petitio* to be remade on *cuero*. The problem, then, is stubborn: If paper was widely used in the intermediary stages of scientific production, like I am suggesting, then it makes sense that we would have scant documentary evidence for the documents that would show it. This is especially the case once we move towards the period of Christianization in the peninsula, where the use of paper is relegated to this kind of secondary use, and it loses all prestige.[22]

It is known that, thousands of unstudied manuscripts notwithstanding, the study of medieval Islamicate history is characterized by a relative lack of documentary evidence,[23] (see Paul, 2015) and the contemporary literary evidence we have is sometimes advised to be taken with caution. This is especially the case with the history of al-Andalus, which due to the "reconquest" and the tribulations associated with it, presents a "practically non-existent" documentary record—a "documentary mass extinction" (Manzano, 2006). This situation is all the more prominent in the scientific case. A somewhat recent collection of essays on available

[22] This may serve as a negative to Rustow's proposal (2020) about the role of prestige in the spread of paper across the Islamicate world. At least in medieval Castile paper was used despite, rather than because of, the governmental attitudes towards it. The situation is different in the Crown of Aragon, mostly due to its local paper industry (Xàtiva is located in the region), which made the product a competitive commodity and hence motivated the Crown to its use.

[23] King (1996) recounts how, on Ibn al-Qifṭī's account, the eleventh-century Egyptian astronomer Ibn al-Sanbadī visited the Cairo library when he learned its manuscripts were being organized. He discovered an extensive collection of 6,500 texts covering astronomy, mathematics, and philosophy. None of those manuscripts are in the library of Cairo today.

documents in the Islamicate world (where al-Andalus occupies a significant portion of the collection) includes no reference to scientific documents, (Sijpesteijn, 2006) and the majority of documents we are aware of consist of *final* products—mostly in the form of books. Despair seems reasonable: What should we even expect for intermediary documents?

If the prospect of finding explicit intermediary documents does not seem too hopeful, the available literature of the period does not improve things much. Take as an example the excerpt from al-Zarqālī that we looked at earlier, where he recounts storing observations for 25 years. What did him and the rest of his collaborators do with those documents? How did they store them? What did they do with them once they were no longer useful? This problem of intermediary scientific documents opens up new considerations of archival practices, (see Paul, 2015; Delsalle and Procter, 2017) and goes beyond the threat of assimilating Islamic archival practices to European ones. (Pickett and Sartori, 2019) Intermediary documents present the characteristic that they are designed to be discarded; or, to be more precise, they are archived only *while* the practitioner takes them to fulfill the task at hand. But, after the task is completed (or the book written), those documents are discarded.

But it would be misleading to say that discarded documents do not have a place in the archival literature. The Cairo Geniza papers are, of course, the paradigmatic case, which in Hirschler's words (2016) presents a case of "counter-archival practices": documents that were stored precisely because they were discarded. In fact, a study of the digitalized documents available in the Geniza Lab database offers a number of scrap papers that fit very well with the ideas on computation I described in the previous section. (Figure 6) Nevertheless, none of the identified documents in the Geniza are of interest for the period and region of interest in this paper.[24]

This whole situation makes sense in the context of the hypothesis we're working with. Part of the absence of these documents can be explained by the fact that they were destined to be sold or discarded, or reused until they were discarded. We should speak, in el-Leithy's (2011) words, as a "life-cycle" of scientific scratch work that saw a reinvigoration with the introduction of a new, inexpensive material. In this sense, one way of looking at their absence in the archives is not as a bug, but as a feature—reflecting the beginning of a transition from the parchment-based reproduction of scientific knowledge to a paper-based practice that expanded the space of intermediary production. It is precisely in this expansion where the key implications of my argument appear, and that I tried to sketch in the previous section. This is particularly patent in the scientific case, where intermediary documents provide a window to a science caught in the middle of its production.

[24] There is one piece (see JRL SERIES A 697 1 / 1 leaf, verso) of astronomical tabulation that stems from Abraham Zacut's 1513 tables (Goldstein and Chabás, 2008), but even assuming the document was written in the peninsula, the dates are off by at least three centuries.

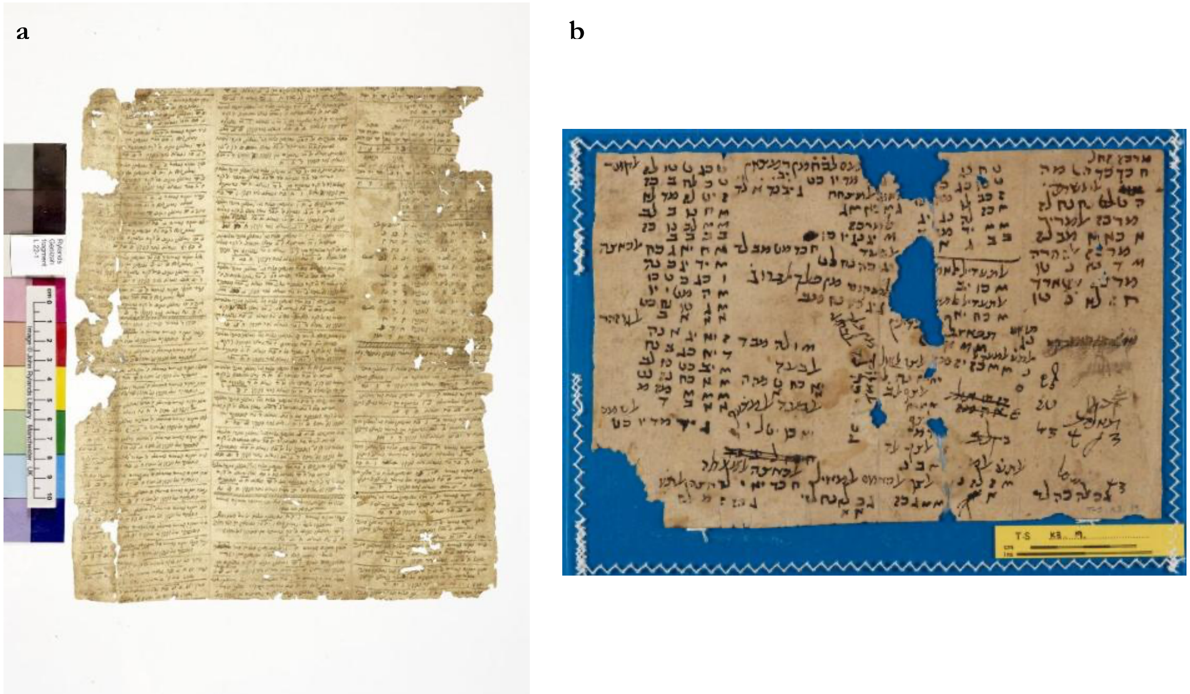

Figure 6. a) Table of astronomical calculations. JRL Series L 22 1 / 1 leaf, recto. **b)** Astronomical or astrological jottings in Judaeo-Arabic. T-S K3.19 1r. Both in paper.

While this realm of scientific practice is certainly challenging from a historiographical point of view, it is well-known that absence of evidence does not equal evidence of absence. And thus, the touchstone of this particular process involved in the introduction of the new writing support does not lie so much in the potential extant documents, but in the inevitability of the problem itself. If a new writing material was in use—and I hope I was able to give some reasonable suspicion it did—then what effects did this have? How did it insert itself into already existing practices, and what new practices did it help engender? The case of medieval Iberia is only one of a growing literature on the matter. (Dover, 2021)

5. Conclusion

Writing supports, I argued, can constrain how information is accessed, processed, and distributed. This is what the example of paper was ultimately meant to show. Many historians have assumed that paper, by providing a cheaper medium for writing, led to transformation in medieval science, but we were missing some explanation as to how this occurred. The case of the Iberian Peninsula aimed at giving a concrete proposal for how this may have happened. A cheaper writing support meant cheaper books (hence more accessible information), as well as an expanded infrastructure for the writing of subsidiary documents involved in the scientific process, and an expanded capacity to off-load the cognitive demands of laborious computations.

But, of course, this is still only a hypothesis. As Rustow (2020) has emphasized, the relatively small documentary evidence of the period pales in comparison with the number of manuscripts that are yet to be studied. How this may scale to intermediary scientific documents is yet to be determined. But—and this is the key point—that does not undermine the problem itself. As the examples from the Geniza show, the use of intermediary computational documents is undeniable. The question, then, is not whether paper had or did not have intermediary use, but what function it had. The excerpt from al-Zarqālī was a prime example of that, where the allusion to scientific production suggested, but simultaneously concealed, questions about the practices necessary for that production.

To really take up this problem in earnest we need to think of scientific production in a different way. The history of writing supports provides a privileged window into the material conditions of information processing, itself a crucial part of scientific production. Does what's written need to be erased before I inscribe new information in my surface? Can the soil around my city provide the raw materials for an affordable writing material? By conditioning the infrastructure of information processing, these problems ultimately affect the practitioners of any discipline—scientific or not—that required the compilation of observations, computations, and communication. These practices were very much alive during the Middle Ages, and understanding how they operated would greatly enhance our understanding of scientific production at a crucial stage of its history.

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